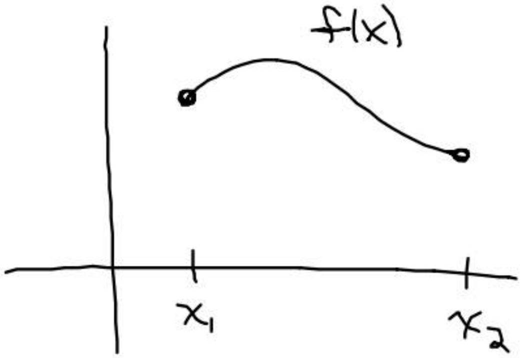
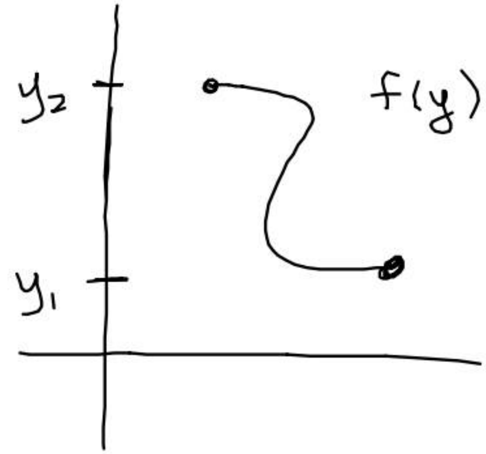
Arc Length Formulas:



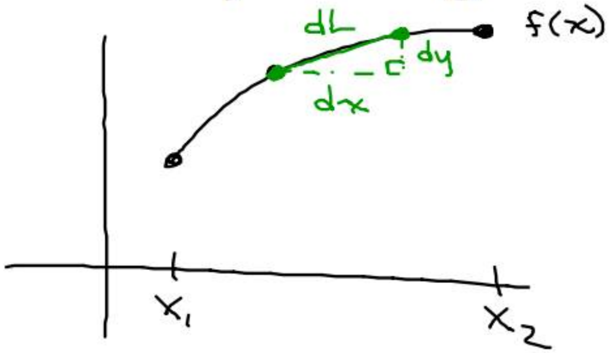
$$\boxed{L = \int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx}$$



$$\boxed{L = \int_{y_1}^{y_2} \sqrt{1 + \left[f'(y)\right]^2}\, dy}$$

★ f(x) AND f(y) MUST BE DIFFERENTIABLE

Derivation of Arc Length Formula:



$$(dL)^2 = (dx)^2 + (dy)^2$$

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$L = \int \sqrt{(dx)^2 + (dy)^2}$$

$$L = \int \sqrt{(dx)^2 + (dy)^2}\ \frac{dx}{dx}$$

$$= \int \sqrt{\frac{(dx)^2 + (dy)^2}{(dx)^2}} \cdot dx$$

$$= \int \sqrt{\frac{(dx)^2}{(dx)^2} + \frac{(dy)^2}{(dx)^2}}\ dx$$

$$= \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

Ex: $y = x^{3/2}$ FROM $(1,1)$ TO $(2, 2\sqrt{2})$

$y' = \frac{3}{2}x^{1/2}$

$$L = \int_1^2 \sqrt{1 + \left(\frac{3}{2}x^{1/2}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{9}{4}x}\, dx$$

$u = 1 + \frac{9}{4}x$

$du = \frac{9}{4}dx$

$$= \frac{4}{9}\int_{13/4}^{22/4} u^{1/2}\, du$$

$$\frac{4}{9}\cdot\frac{2}{3}u^{3/2}\Big]_{13/4}^{22/4}$$

$$\frac{8}{27}\left(\left(\frac{22}{4}\right)^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right)$$

$$\boxed{\frac{1}{27}\left(22\sqrt{22} - 13\sqrt{13}\right)}$$

b) w|r|t y: $y = x^{3/2} \Rightarrow x = y^{2/3} \Rightarrow x' = \frac{2}{3} y^{-1/3}$

$$L = \int_1^{2\sqrt{2}} \sqrt{1 + \left(\frac{2}{3} y^{-1/3}\right)^2}\, dy$$

$$= \int_1^{2\sqrt{2}} \sqrt{1 + \frac{4}{9} y^{-2/3}}\, dy$$

$$= \int_1^{2\sqrt{2}} \sqrt{y^{-2/3}\left(y^{2/3} + \frac{4}{9}\right)}\, dy$$

$$\int_1^{2\sqrt{2}} \cancel{y^{-1/3}} \sqrt{y^{2/3} + 4/9}\, dy$$

$u = \cancel{y^{2/3}} + 4/9$

$du = 2/3\, \cancel{y^{-1/3}}\, dy$

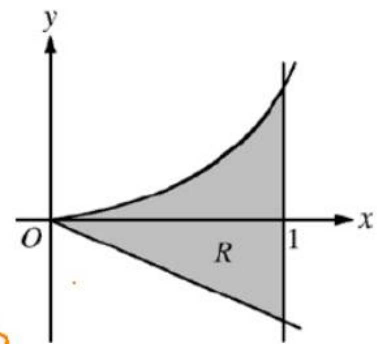
$$\frac{3}{2} \int_{13/9}^{22/9} u^{1/2}\, dy$$

$$\cancel{\tfrac{3}{2}} \cdot \cancel{\tfrac{2}{3}} u^{3/2} \Big]_{13/9}^{22/9} = \left(\frac{22}{9}\right)^{3/2} - \left(\frac{13}{9}\right)^{3/2}$$

$$\boxed{= \frac{1}{27}\left(22\sqrt{22} - 13\sqrt{13}\right)}$$

Question 5

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

$u = x^2$

$du = 2\cancel{x}\,dx$

a) $\int_0^1 xe^{x^2} + 2x\,dx$

$\int_0^1 \cancel{x}e^{x^2}dx + \int_0^1 2x\,dx$

$\frac{1}{2}\int_0^1 e^u\,du + x^2\Big]_0^1$

$\boxed{\frac{1}{2}(e-1)+1}$

$\frac{1}{2}e - 1/2 + 1$

$\frac{1}{2}e + 1/2$

$\frac{1}{2}(e+1) = \frac{e+1}{2}$

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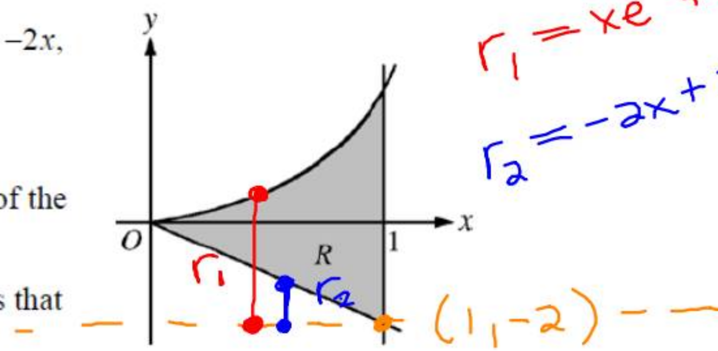

$r_1 = xe^{x^2} + 2$

$r_2 = -2x + 2$

b) $\boxed{V = \pi \int_0^1 r_1^2 - r_2^2 \, dx}$

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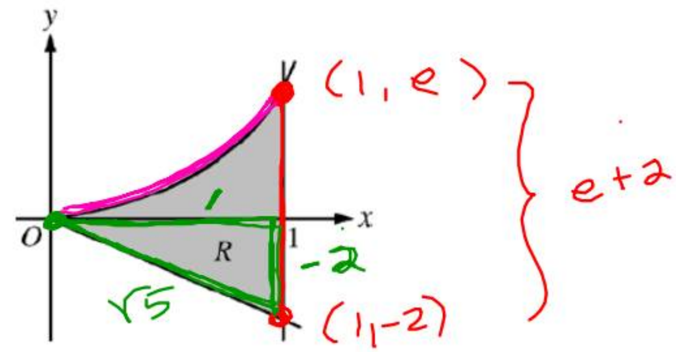

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$y_1 = xe^{x^2}$

$\frac{dy_1}{dx} = x \cdot 2xe^{x^2} + e^{x^2}$

$= 2x^2e^{x^2} + e^{x^2}$

$$\boxed{L = e + 2 + \sqrt{5} + \int_0^1 \sqrt{1 + \left(\frac{dy_1}{dx}\right)^2}\, dx}$$

HW: CHAPTER 7 AP PACKET

15, 30, 61-69 ODD, 70, 71